Soliton interaction near the zero-dispersion wavelength

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A comprehensive analysis of the soliton interaction near the zero-dispersion wavelength when higher-order dispersion comes into the play is presented. It is shown that the interaction process depends critically on the existence of the resonance radiation because that radiation cannot be separated from the soliton in time. We propose to exploit a proper phase modulation to stabilize the positions and the amplitudes of a soliton train provided that third-order dispersion is weak and no resonance radiation occurs. Moreover, we show that the resonance radiation may be absorbed by a bandwidth-limited amplifier regardless of the bandwidth. Thus, an appreciable stabilization of the pulse separation as well as the amplitudes can be achieved. The applicability of a convenient first-order perturbation approach to describe correctly the evolution of pulses near the zero-dispersion wavelength is studied in detail.

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I. INTRODUCTION

It is now a common belief that optical solitons, that may exist in fused silica fibers beyond the zero-dispersion wavelength (ZDW), will play an important role as information carriers in future high-speed communication systems. The fundamental mechanism of soliton formation, namely, the balanced interplay of linear group velocity dispersion (henceforth termed as dispersion) and nonlinearily induced self-phase modulation, is well understood [1,2]. What makes solitons particularly attractive for applications in transmission lines is their remarkable robustness [3]. The major reason for that robustness is that the wave numbers of the solitons are separated from those of the dispersive waves. Therefore, linear waves cannot be in resonance with the soliton, and energy cannot be exchanged provided that no perturbation with just that relevant wave number acts [4,5].

Since the early days of the experimental investigation of optical solitons the reduction of the power needed to create a fundamental soliton has been a pivotal question. This power is directly proportional to the dispersion. Because of the large dispersion of standard fibers at the wavelength of minimal losses ($\lambda = 1.55$ μm) an evident idea for power reduction was a shift of the operational wavelength towards the ZDW [6–8] which is $\lambda = 1.31$ μm. In doing so an additional term in expanding the wave number of the fiber mode $k(\omega)$ around the central frequency ω_0, namely, the third-order dispersion (TOD) term, had to be taken into account. Hence, there was a certain interest in understanding the effect of TOD on the existence and stability of the soliton [5,9–17]. Because the then relevant modified nonlinear Schrödinger equation (NLSE) is not integrable perturbation or numerical methods had to be applied. It was discovered that the pulses (or solitary waves) near the ZDW have a soliton-like shape, but radiate continuously [7]. It was shown that this radiation consists of different spectral components [14,17] where the most prominent is called resonance radiation. This resonance radiation is created beyond a certain threshold of TOD and radiates into the normal dispersion regime where the spectral shift is inversely proportional to TOD [7,11]. Because of the conservation of momentum (or the spectral center of mass) the pulse is shifted further into the anomalous dispersion regime and generates its own dispersion [5,11]. Below the threshold the solitary wave behaves similar to a Schrödinger soliton where the TOD presents an nonsymmetric perturbation to the NLSE and evokes a nonzero velocity [6,18], i.e., the soliton slows down. Moreover, very early it was proposed to use propagation near the ZDW to reduce the soliton interaction [19] that may eventually lead to the coalescence of two solitons. The reason for this reduction is the decay of the two-soliton bound state into two individual solitons of unequal amplitude and velocity [20]. Due to the introduction of dispersion-shifted and flattened fibers as well as erbium-doped fiber amplifiers some years ago the main interest in soliton transmission has clearly turned to the 1.55 μm region where TOD may be neglected provided that picosecond pulses are used.

But, very recently a renewed interest in studying the effects evoked by TOD has been emerging, having essentially three reasons. Firstly, the improvement of the performance of praseodymium-doped fiber [21] and semiconductor amplifiers that operate around 1.3 μm makes this wavelength regime an interesting alternative. Secondly, the use of femtosecond pulses which are generated routinely nowadays may require the inclusion of TOD effects together with self-steepening and intrapulse Raman scattering [22]. Last but not least, the modeling of pulse generation in different types of mode-locked lasers has shown that TOD plays a prominent role [23,24].

Because the effect of TOD on single soliton propaga-

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tion is essentially understood [5,12–18] the main interest is focused on a fundamental understanding of soliton interaction when TOD comes into the play. It is well appreciated that, besides the Gordon-Haus jitter [25] and acoustic effects [26], the soliton interaction significantly contributes to the limitation of the capacity of future transmission lines. Hence, many proposals, e.g., as bandwidth-limited amplification [27,28], the introduction of sliding filters [29], nonlinear amplification [30], the usage of out-of-phase solitons [31,32], and phase modulation [33–36], have been made to reduce the soliton interaction and to stabilize their propagation. Apart from a few papers [37–39] these investigations are restricted to those wavelengths and/or pulse widths where TOD can be disregarded. Among other things the combined effect of a weak TOD, phase modulation and bandwidth limited amplification on single soliton propagation was analyzed recently [40]. Hence, it might be interesting to study the soliton interaction systematically in the presence of TOD as it depends on various initial conditions (separation, phase, amplitude) as well as on the effects listed above.

As long as TOD is weak, where this remains to be specified by deriving the corresponding pulse data, one can expect that the Schrödinger solitons keep their very identity (no resonant radiation) and acquire only a nonvanishing velocity. This effect might be used to separate the solitons in the time domain from the radiation [41], generated by the excess gain of the amplifiers and TOD, in order to avoid soliton instability [30,42]. Moreover, one can expect that perturbation approaches that use Schrödinger solitons as trial functions, e.g., the Karpman-Solov'ev approach (KSA) [43], will yield a reasonable description of the interaction process because only minor radiation is created by TOD.

Although one attempts usually to keep the TOD within this limit it might sometimes be necessary to increase it to get, e.g., a velocity that is sufficiently large to separate solitons and amplifier radiation. But, this will ultimately lead to the generation of the resonance radiation. Conventional perturbation methods will fail to describe the interaction process properly, at least beyond a certain strength of TOD.

Very recently, it was proposed to use the emitted resonance radiation to mediate the interaction between an array of solitons which may eventually form a stable bound state [44]. These solitons can then be stably pinned by their common radiation field provided that their separation exceeds a critical value. These results remain to be double-checked by numerical methods.

Other authors [45] claim that the main reason for the appearance of resonance radiation is the absence of symmetry in a "one-hump" solution. They found a "double-hump" solution that forms a bound state without emitting radiation. Until now there is no evidence that this solution can be excited and that it propagates stably.

The aim of the present paper is twofold. Firstly, we intend to identify the influence of both weak and strong TOD on the soliton interaction where particular emphasis is paid to the analysis of the radiation generated. Furthermore, we demonstrate how phase modulation and bandwidth-limited amplification can be used to stabilize amplitudes and positions for weak and strong TOD, respectively. We show the case of N-soliton interaction which is identified to be fundamentally different from the two-soliton case. Secondly, we intend to show that the Karpman-Solov'ev approach proves as a reasonable, versatile and powerful tool to describe two-soliton interaction in the presence of TOD. It yields the famous decay of the bound state and allows the straightforward inclusion of various initial conditions as, e.g., pulse separation and phase difference, and perturbations as, e.g., bandwidth-limited amplification (BLA) and phase modulation. Moreover, we are identifying the limits of its applicability with increasing TOD.

The paper is organized as follows: After a concise mathematical formulation of the problem we study the soliton interaction in the presence of TOD in Sec. II and its reduction by a proper phase modulation in Sec. III. The combined effect of TOD and BLA on the interaction process is investigated in Sec. IV where the role of the resonance radiation is investigated in detail. In the case of N-soliton interaction we have to rely on pure numerical studies. In the Appendix we discuss the limits of applicability of the perturbation approach.

II. THIRD-ORDER DISPERSION AND SOLITON INTERACTION

The evolution of the normalized pulse envelope $\Psi(z,t)$ in an optical fiber may be described by the perturbed nonlinear Schrödinger equation

$$i\frac{\partial\Psi}{\partial z}+\frac{1}{2}\frac{\partial^2\Psi}{\partial t^2}+|\Psi|^2\Psi=i\gamma\frac{\partial^3\Psi}{\partial t^3}+R(\{\Psi\}) , \tag{1}$$

where R stands for any perturbation (except TOD) as, e.g., loss, bandwidth-limited amplification, phase modulation and filtering, and is set in this section to zero and specified later on. The normalized time in the moving frame of the pulse t and the distance z are related to the variables T and Z as

$$z=\frac{|k_2|}{T_0^2}Z=\frac{Z}{Z_0} , \quad t=\frac{T-k_1Z}{T_0} , \tag{2}$$

where T_0 can be related to the pulse length (T_{FWHM}) by $T_{\mathrm{FWHM}}=1.76T_0$, Z_0 is the dispersion length, $k_1=\partial k/\partial\omega|_{\omega_0}$ the inverse group velocity of the pulse and $k_2=\partial^2 k/\partial\omega^2|_{\omega_0}$ the group velocity dispersion at the mean frequency. The normalized envelope Ψ is related to the slowly varying envelope $A(z,t)$ of the fiber mode as

$$\Psi=T_0\left[\frac{\omega_0}{c}\frac{n_2}{|k_2|A_{\mathrm{eff}}}\right]^{1/2}A , \tag{3}$$

where n_2 and A_{eff} are the nonlinear coefficient and the effective core area of the fiber, respectively. The strength of TOD is reflected by the normalized quantity

$$\gamma=\frac{k_3}{6|k_2|T_0} , \quad \text{with } k_3=\left.\frac{\partial^3 k}{\partial\omega^3}\right|_{\omega_0} . \tag{4}$$

In order to get some impression about realistic values for

γ one can use an approximate formula [7], that holds for a standard fiber, and obtains for a 1 ps pulse full width at half maximum (FWHM) and a wavelength offset $\Delta\lambda=0.005$ μm from ZDW a TOD of $\gamma=0.05$ being slightly above the threshold for the generation of resonance radiation ($\gamma=0.045$ [14]).

Our first aim consists in studying the interaction of solitons when only TOD is present. Because (1) is not integrable we have to use perturbation approaches or numerical methods as the beam propagation method (BPM) to find the respective solitary wave solutions. As far as the numerical solution is concerned we have used a standard split-step fast Fourier transform BPM [46]. There are various powerful perturbation approaches which rely on different assumptions and have their particular merits and disadvantages. The well-established variational approach is very versatile because it allows for the inclusion of independent amplitudes and widths as well as a chirp acquired during the propagation. But this approach is restricted to perturbation $R(\{y\})$ that are Hamiltonian and allow for the introduction of an Lagrangian. Because this is not the case for very prominent perturbations as, e.g., losses and bandwidth-limited amplification we use here the Karpman-Solov'ev approach. Only very recently this approach was applied to study the effect of TOD on soliton propagation and interaction [39]. The main idea of KSA is to choose a superposition of Schrödinger solitons as a trial function and to derive ordinary differential equations for the soliton parameters which are driven by the interaction as well as the perturbation terms. The two-soliton trial function may be written as

$$\Psi(z,t)=\sum_{n=1}^{2} 2\nu_n \operatorname{sech}[2\nu_n(t-\xi_n)] \times\exp\{i[2\mu_n(t-\xi_n)+\delta_n]\} , \tag{5}$$

where $2\nu_i$, $2\mu_i$, ξ_i, and δ_i, $i=1,2$ are the amplitudes, frequencies, positions and phases of the pulses, respectively. It is evident from (5) that all processes that are generating radiation (dispersive waves) and change the pulse shape are not accurately described by KSA. Because it is well known that TOD may do both [7,14] the limits of applicability of KSA have to be checked very carefully (see the Appendix). Following the procedure outlined in [43] we end up with the system of differential equations for the pulse parameters as

$$\frac{d\mu_n}{dz}=(-1)^n 16\nu^3\exp(-2\nu r)\cos\Phi , \tag{6}$$

$$\frac{d\nu_n}{dz}=(-1)^n 16\nu^3\exp(-2\nu r)\sin\Phi , \tag{7}$$

$$\frac{d\xi_n}{dz}=2\mu_n+4\nu\exp(-2\nu r)\sin\Phi+4\gamma(\nu_n^2+3\mu_n^2) , \tag{8}$$

$$\frac{d\delta_n}{dz}=2(\nu_n^2+\mu_n^2)+8\nu\exp(-2\nu r)(\mu\sin\Phi+3\nu\cos\Phi) +16\gamma\mu_n(\mu_n^2-\nu_n^2) , \tag{9}$$

where $\nu=(\nu_1+\nu_2)/2$, $\mu=(\mu_1+\mu_2)/2$, $r=\xi_1-\xi_2$, $\Psi=\delta_2-\delta_1$, and $\Phi=2\mu r+\Psi$. In deriving (6)–(9) it had to be assumed that the fluctuations in the amplitudes and velocities are small with respect to their averaged values ($|\mu_1-\mu_2|\ll\mu$, $|\nu_1-\nu_2|\ll\nu$) and the separation of the two pulses is not too small ($\nu r\gg 1$). It is evident from (6) and (7) that the averaged amplitude and velocity are conserved quantities. Moreover, Eq. (8) reflects that the solitons acquire a velocity due to TOD.

In order to make sure that the results derived by using the system (6)–(9) are reliable we have solved that system for various situations and double-checked the results by using the BPM. Details are presented in the Appendix. The conclusion that we can draw from these studies is that the perturbation approach may be applied for fairly large initial separations [$r(0)\geq 8$, corresponding to about 4.5 pulse widths] and TOD coefficients $\gamma\leq 0.05$, being slightly beyond the threshold where resonance radiation is generated. Within these limits the agreement becomes almost perfect if the pulses are initially out of phase by π.

Figure 1 shows that the soliton bound state decay [20], caused by TOD as a nonsymmetric perturbation, can be reasonably described by using KSA. Both the evolution of the positions and the amplitudes follow closely the BPM results except at the point where the separation is smallest and the assumptions made to derive the KSA equations are violated. Note that the different amplitudes of the single solitons emerging eventually are very close to those derived by the inverse scattering transform [20]. Very early it was proposed to exploit just that TOD triggered effect to avoid the coalescence of two in-phase solitons. Unfortunately, this concept cannot be extended towards N-soliton interaction. The BPM calculations reveal (see Fig. 2) that initially the behavior is similar to the two-soliton case and can be interpreted as the decay of the N-soliton bound state. Depending on the different velocities acquired the pulses continue to separate [Fig. 3(a)] or coalesce after some propagation distance [Fig. 3(b)]. Hence, we may conclude that TOD does not suffice to stabilize a multisoliton train.

Until now we dealt with situations where the TOD parameter was only slightly above the threshold value ($\gamma=0.045$) for the generation of resonance radiation. It might be interesting to investigate the case where that parameter is considerably larger, although one should keep in mind that, e.g., $\gamma=0.2$ would require 250 fs pulses for a deviation of 5 nm from the ZDW. The numerical simulation shown in Fig. 4 displays both the decay of the bound state, where the pulses acquire different group velocities and amplitudes [see also Fig. 5(a)], and the fairly strong emission of radiation. Following the idea of Wen and Chi for the single pulse propagation [14] we analyze the spectrum of the interacting pulses, the shape of which is shown in Figs. 5(a) at $z=8\pi$. In Fig. 5(b) the complete spectrum is displayed where the strong peak of the resonance radiation (q_r) can clearly be identified. Moreover, we have studied the spectral content appearing in different time slots, shown in Figs. 5(c)–(e). It is clearly seen that the component in front of the leading edge of the first pulse (q_p) radiates with two frequencies where one of them is close to $\Omega=0$ [$\Omega=(\omega-\omega_0)T_0$] [see Fig. 5(c)]. Behind the trailing edge the radiation contains the strong resonance contribution emitted from both pulses and a weak part (q_n) slightly below $\Omega=0$ [see Fig. 5(d)]. The time slot covering the region where the two pulses are situated contains a small contribution of the resonance radiation stemming exclusively from the leading pulse Fig. 5(e). By using the dispersion relation of the linear radiation

$$\kappa_{\rm lin}=-\frac{\Omega^2}{2}+\gamma\Omega^3 , \tag{10}$$

we find the deviation $\Delta_{\rm lin}$ from the normalized inverse group velocity, being proportional to k_1 [see (1)], to be

$$\Delta_{\rm lin}=\frac{\partial\kappa_{\rm line}}{\partial\Omega}=-\Omega+3\gamma\Omega^2 . \tag{11}$$

On the other hand, it was shown that the change in the inverse group velocity of the soliton Δ_s beyond the first-order approximation is [38]

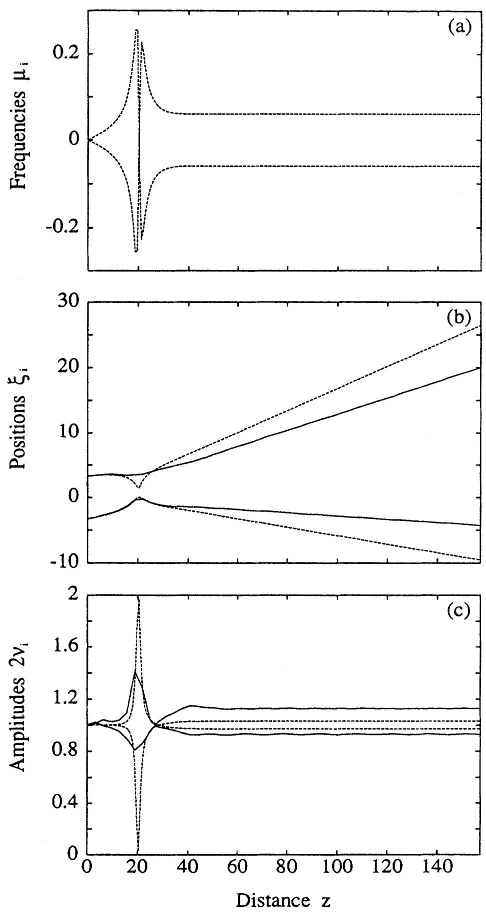

FIG. 1. Decay of the two-soliton bound state-BPM versus perturbation approach $\gamma=0.05$ (moderate TOD); initial separation $r(0)=6.5$; solid line—BPM; dotted line—perturbation approach. Unless otherwise specified the modulus $|\Psi|$ of the amplitude is shown in all figures. We have used dimensionless quantities, introduced in the text, in all figures. (a) Evolution of frequencies μ_i, $i=1,2$; (b) evolution of the positions ξ_i, $i=1,2$; and (c) evolution of the amplitudes $2\nu_i$, $i=1,2$ as a function of the distance.

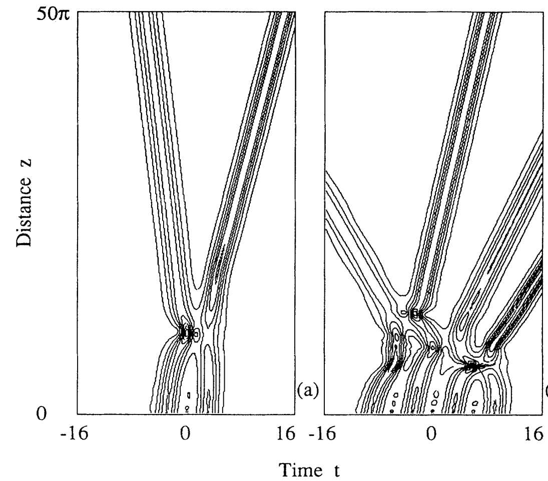

FIG. 2. Similarity between the decay of the (a) two-soliton and (b) multisoliton bound state; $\gamma=0.05$ (moderate TOD), initial separation $r(0)=6$.

$$\Delta_{\rm sol}=\frac{\partial\kappa_{\rm sol}}{\partial\Omega}=\gamma+12\gamma^3 . \tag{12}$$

Equation (11) implies that the radiation contributions q_p and q_n are almost stationary. Thus, both separate in time from the soliton [see (12)].

The frequency shift of the dominant resonance radiation q_r is given by the matching of the NLSE soliton and the radiation wave vector and amounts to $\Omega_r=1/2\gamma+2\gamma(2\nu)^2$ [16]. By using (11) we get

$$\Delta_r=\frac{\partial k_{\rm lin}}{\partial\Omega}\bigg|_{\Omega_r}=\frac{1}{4\gamma}+4\gamma(2\nu)^2+O(\gamma^3) . \tag{13}$$

Hence, this radiation always propagates in the wake of the soliton that generates it. Moreover, by using (12) and (13) it is very easy to show that the larger the TOD the smaller the group velocity difference between soliton and radiation. On the other hand, the frequency shift, also

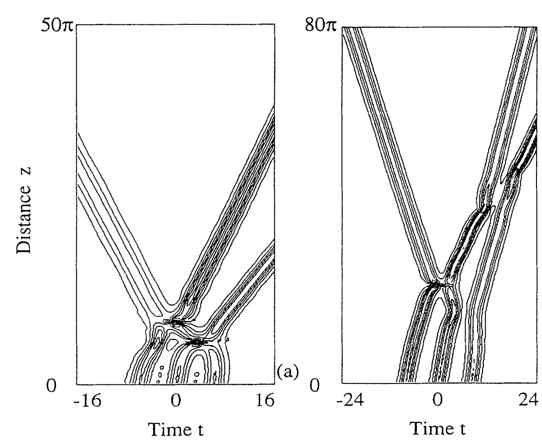

FIG. 3. Decay scenarios of the three-soliton bound state for different initial separations; $\gamma=0.05$ (moderate TOD); (a) $r(0)=6$—continuous separation; (b) $r(0)=8$—initial separation of subsequent coalescence.

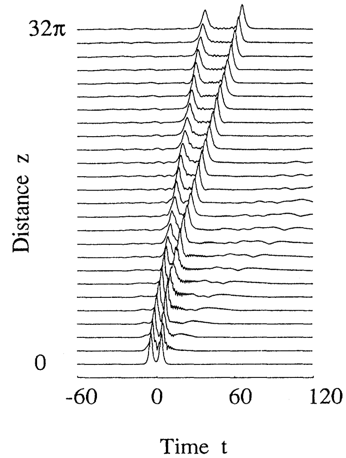

FIG. 4. Two-soliton interaction and the emission of resonance radiation for large TOD ($\gamma=0.2$); initial pulse separation $r(0)=8$.

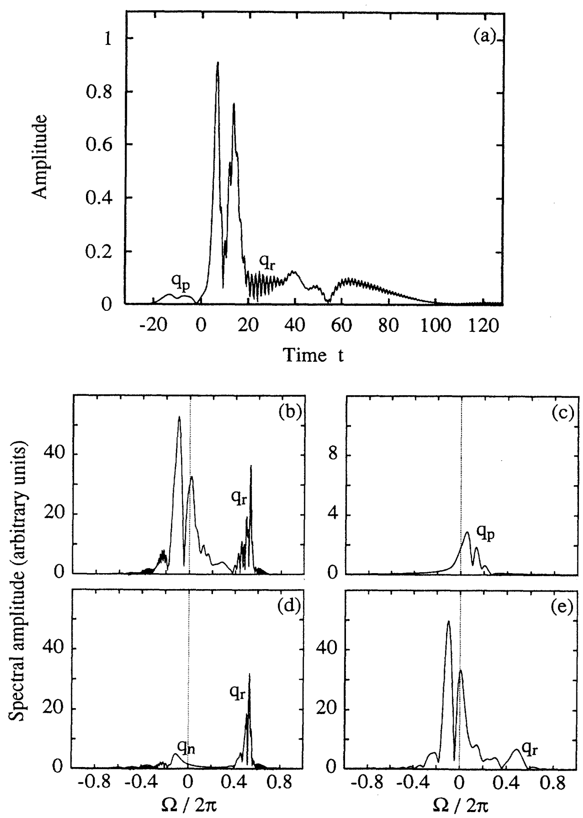

FIG. 5. Envelope and spectrum of two interacting pulses as well as the dispersive waves at $z=8\pi$ and with $\gamma=0.2$: (a) envelope $|\Psi|$; (b) spectrum of the entire field shown in (a); (c) spectrum of the dispersive waves in front of the leading edge ($t \leq 0.4$); (d) spectrum of the dispersive waves behind the trailing edge ($t \geq 19.4$); (e) spectrum of the pulses and the dispersive waves in between them ($0.4 < t < 19.4$).

seen in Fig. 5(b), 5(d), and 5(e) implies that, at least, the resonance radiation might be removed by appropriate filtering. We will come back to this issue in Sec. IV. The dominant role of the resonance radiation is displayed in Fig. 6 where the relative energy of the different radiation components is shown for strong TOD not considered in [14]. It is worth noting that until very recently particular emphasis is paid to the detailed study of the resonance radiation [12–17]. Very recently, a complete analytical description of the different parts of the emitted radiation was given by Elgin, Brabec, and Kelly [17].

Due to the appearance of dispersive waves it is no surprise that there is an appreciable discrepancy between the numerical and the KSA results. This is shown in Fig. 7 for the evolution of the pulse separation. One might be inclined to attribute this discrepancy exclusively to the improper trial function chosen but it will turn out later that regardless of the radiation the inclusion of higher order perturbation terms might be required [47].

At this point it is interesting to discuss in more detail the idea of the TOD-mediated soliton bound states put forward in [44,45]. Provided that we launched two NLSE solitons to the fiber we found no numerical evidence that the radiationless "two-hump solution" [45] evolves, neither for a weak nor for a strong TOD. On the contrary Malomed's idea [44] relies on the emission of radiation. Hence, one would expect these bound states to appear only beyond the threshold of resonance radiation, i.e., for strong TOD. He assumed that the solitons riding on top of a common radiation "substrate" are affected by an effective pinning potential. A bound state can then be formed when the pinning is stronger than the mutual attraction caused by the interaction forces. The separation between the bound solitons may take an arbitrary value larger than a critical one, at which the attraction forces are equal. This critical separation was estimated to be $r(0)_{\rm cr}\approx\pi/2\gamma$ [44] corresponding approximately to the separation used in Fig. 4. Two assumptions have been made to derive the critical distance for the bound state to exist, namely, that the interaction potential is not affected by TOD and that both pulses are affected by a common "substrate." The former is certainly violated, but probably not very critical. As for the second one, it can be clearly identified in inspecting Figs. 4 and 5 that the resonance radiation is asymmetric, always situated behind the

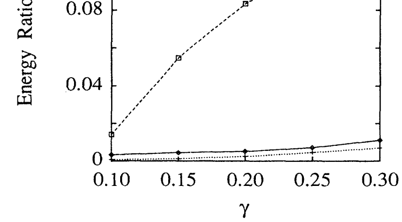

FIG. 6. Ratio of the energies of the different dispersive waves to the initial soliton energy as a function of TOD(γ) for a single pulse; $z=8\pi$; q_r (dashed), q_n (dotted), and q_p (solid).

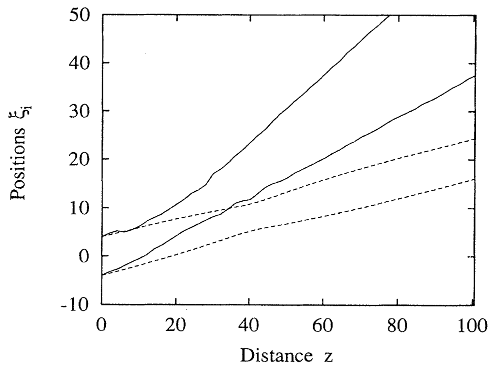

FIG. 7. Evolution of the positions of both pulses as a function of the propagation distance: initial parameters: $r(0)=8$; $\nu_i=0.5$; $\mu_i=0$, $i=1,2$; $\gamma=0.2$ (strong TOD) dashed line—KSA; solid line—BPM.

trailing edge of the pulses [see (12)]. Hence, the leading pulse is not affected by the same radiation substrate as the trailing pulse. In order to clarify the situation we performed a numerical experiment. In Fig. 8(a) the propagation of two pulses in the presence of TOD is seen. Obviously, no bound state is formed because of the absence of radiation in front of the leading pulse. This radiation can be artificially generated if we decrease the time window and use periodic boundary conditions. Figure 8(b) displays this situation. There is some evidence that such a common substrate is formed and that the pulses are stuck together. Hence, we conclude that the prediction of a bound state of two pulses to appear cannot be expected in a realistic experiment, but the situation appears differently if a train of pulses on a soliton ring laser is concerned. Then, the common substrate might exist for all pulses except the leading one.

III. THIRD-ORDER DISPERSION AND PHASE MODULATION

One of the results of the preceding section was that a weak TOD can only prevent the coalescence of two solitons, but generally not that of N solitons. Moreover, because the pulses acquire different velocities it is not possible to stabilize their positions. But, just this would be the ultimate goal in a transmission link in order to increase the capacity and the reliability of the information transmitted. In recent papers [35,36] it was shown, but without TOD, that a phase modulation applied to the pulse train can stabilize the positions of the individual pulses. The idea is close to that just discussed, namely, the introduction of a periodic potential that may capture the pulses. But here this potential stems from the distributed action of periodically aligned phase modulators. If their spacing distances are much less than the dispersion length their action can be described in adding an perturbation term [35]

$$R(\{\Psi\})=\alpha\cos(\tilde{\Omega}t)\Psi\ , \tag{14}$$

where α and $\tilde{\Omega}$ are the corresponding normalized amplitude and frequency of the modulation, respectively. Because the velocity created by TOD gives the pulses some "kinetic" energy the potential is expected to need some critical modulation depth. Moreover, one can anticipate that the pinning effect of the potential may be eventually that strong that some critical initial separation, that was $r(0)=8$ in the previous investigations, can be reduced. Evidently, the modulation frequency should match to the initial separation. We start with the two-soliton case to understand the underlying physics and proceed then with the N-soliton case. Taking into account (14) the KSA Eqs. (6) and (9) for the frequencies and phases change to

$$\frac{d\mu_n}{dz}=(-1)^n16\nu^3\exp(-2\nu r)\cos\Phi +\alpha\frac{\tilde{\Omega}\Theta_n}{2}\frac{\sin(\tilde{\Omega}\xi_n)}{\sinh\Theta_n}\ , \tag{15}$$

$$\frac{d\delta_n}{dz}=2(\nu_n^2+\mu_n^2)+8\nu\exp(-2\nu r)(\mu\sin\Phi+3\nu\cos\Phi) +16\gamma\mu_n(\mu_n^2-\nu_n^2)-\alpha\Theta_n^2\cos(\tilde{\Omega}\xi_n)\frac{\cosh\Theta_n}{\sinh^2\Theta_n}\ , \tag{16}$$

where $\Theta_n=\pi\tilde{\Omega}/4\nu_n$.

As can be seen from (15) phase modulation affects explicitly the frequency which in turn acts on the position via (8), thus competing with TOD. Hence we may expect that for a fixed TOD a minimal amplitude of phase modulation $\alpha_{\min}$ can be derived being necessary to stabilize both the amplitudes and the positions of two interacting pulses. In what follows we assume that the initial separation is very small $[r(0)=6]$ and determine the minimal amplitude for varying TOD. Moreover, we study the situations where the pulses initially launched to the fiber are in phase $[\Psi(0)=0]$ or out of phase $[\Psi(0)=\pi]$. We use both the KSA and the numerical approach. The criteria for the permitted fluctuations of the separation as well as the relative amplitudes are set to $|\Delta r|\leq 0.5$ and $|\nu_1-\nu_2/\nu|\leq a$, respectively, where a varies for different situations. For in-phase pulses we

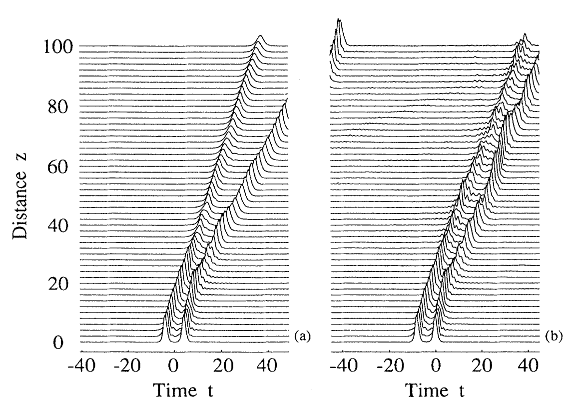

FIG. 8. Numerical experiment: artificial creation of a "common" radiation substrate by decreasing the "time window" in (b); no absorbers were used. Here, the intensity is shown and the parameters are $r(0)=8$ and $\gamma=0.2$. (a) time window $t=1024$; (b) time window $t=90$.

have set $a \leq 0.3$ because the KSA yields fairly large fluctuations whereas for out-of-phase pulses it can be reduced to $a \leq 0.1$. The results are shown in Fig. 9 where the propagation distance was $z = 50\pi$. It is evident that the minimal amplitude of phase modulation has to be larger for in-phase pulses. Moreover, the agreement between the results provided by the perturbation and the numerical approach is much better for out-of-phase pulses because the pulse shapes are fairly well conserved in this case. As it could be anticipated the minimal amplitude has to increase with third-order dispersion because the induced potential has to prevail against the velocity caused by TOD. Although the initial separation is very small the perturbation approach yields reasonable results for out-of-phase solitons. For in-phase pulses considerable changes of the pulse shapes allow merely for a qualitative agreement.

Eventually, we use the results obtained for the two-soliton interaction as a basis to study the evolution of a multisoliton train numerically. We have simulated the cases where the train consists of three and four pulses and the initial separation was $r(0)=6$ or 8. As a representative example we present here the results for the 4-pulse-train with $r(0)=6$. We assume that TOD is weak and no appreciable resonance radiation is created. Successively, we investigate the mere effect of TOD, then add an initial phase shift of π between adjacent pulses, apply only phase modulation and eventually combine all effects. The respective results are depicted in Fig. 10(a)–10(d). It is obvious from Fig. 10(a) that similarly to the two-soliton case the N-soliton bound state decays where the positions change during the propagation. For out-of-phase pulses it can be recognized that the pulse shapes and amplitudes are almost conserved, but a walkoff is left [see Fig. 10(b)]. In both situations TOD prevents pulse collision, but decreases the capacity of the channel due to the walkoff. If we add phase modulation with an appropriate amplitude (see Fig. 9) and a matched frequency $[\tilde{\Omega}=2\pi/r(0)]$ a clear stabilization can be identified. Note, that for in-phase pulses the modulation amplitude has to be larger and a minor fluctuation of the pulse amplitudes is left [see Fig. 10(c)]. For out-of-phase solitons a substantial stabilization of both the amplitudes and positions can be clearly recognized from Fig. 10(d).

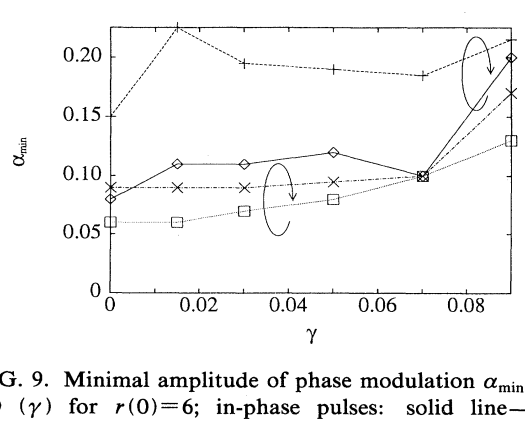

FIG. 9. Minimal amplitude of phase modulation $\alpha_{\min}$ versus TOD (γ) for $r(0)=6$; in-phase pulses: solid line—BPM; dashed line—KSA; out-of-phase pulses: dotted line—BPM; dash-dotted line—KSA.

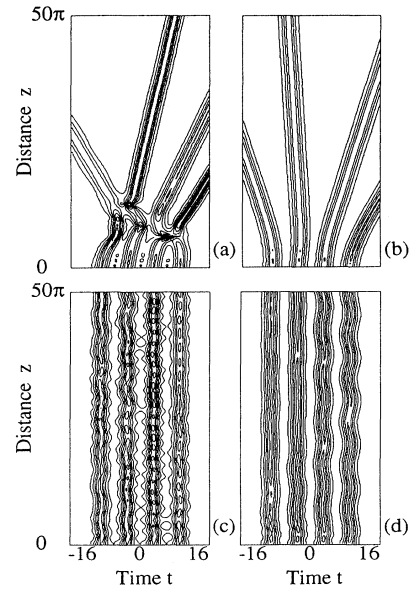

FIG. 10. Effect of phase modulation on multisoliton interaction for moderate TOD ($\gamma=0.05$) and small initial separation $[r(0)=6]$. (a) Decay of the bound state for in-phase $[\Psi(0)=0]$ pulses and $\alpha=0$; (b) repulsion of out-of-phase $[\Psi(0)=\pi]$ pulses and $\alpha=0$; stabilization of the amplitudes; (c) partial stabilization of the positions and amplitudes of in-phase pulses by phase modulation: $\alpha=0.12$; $\tilde{\Omega}=\pi/3$; (d) suppression of interaction and stabilization of both the amplitudes and the positions of out-of-phase pulses by phase modulation: $\alpha=0.08$; $\tilde{\Omega}=\pi/3$.

We may conclude that phase modulation is an appropriate tool to overcome the detrimental effects of soliton interaction and TOD below the threshold for the generation of resonance radiation. Moreover, the idea to exploit a periodic potential to capture the solitons [35] works at least for the case of a deterministic potential not caused by resonant radiation.

IV. THIRD-ORDER DISPERSION AND BANDWIDTH-LIMITED AMPLIFICATION

In long-haul transmission lines solitons need to be periodically amplified after typical distances of 30–40 km to compensate for the absorption losses. Nowadays this is routinely achieved in using lumped erbium-doped amplifiers [48]. In first-order approximation these amplifiers may be described by an inverted two-level system which has a finite bandwidth. It has been shown that so-called "averaged solitons" may propagate provided that the amplifier spacing (Z_a) is much less than the dispersion length (Z_0) and the gain and bandwidth have a definite relation [49–53]. Note, that these "averaged solitons" are asymptotically unstable for very large distances because of the unavoidable interaction with the amplified radiation. Moreover, it was proposed to employ this bandwidth-limited amplification (BLA) to reduce the noise-induced temporal jitter (Gordon-Haus jitter) [54,55] and the soliton-soliton interaction [27,28]. With regard to the latter issue it has been shown that the improvement is only marginal as long as the bandwidth is large [27,28,42]. Decreasing the bandwidth requires an increase of the gain which leads to the so-called soliton instability due to the strong amplification of the low frequency radiation generated by the amplifiers [30,42]. Generally BLA causes two major effects in the frequency domain. It cuts high-frequency radiation [56] and it pulls back the soliton frequency to the center of the filter curve [28]. In view of TOD-triggered effects as, e.g., the generation of different kinds of radiation and the decay of the soliton-bound state it is worthwhile to have a fresh look at BLA. In particular, possible implications with regard to the soliton interaction may attract some interest.

The transfer function $T(\Omega)$ of the lumped amplifiers may be written as

$$T(\Omega)=\frac{\exp(G)}{1+2i\frac{\Omega}{B}}\ , \tag{17}$$

where we have assumed that the center frequency of the amplifier coincides with the mean frequency ω_0 of the pulses. We have then $\Omega=(\omega-\omega_0)T_0$ and $B=\omega_f T_0$ where B and ω_f are the normalized and the real amplifier bandwidth, respectively, and $\exp(G)$ is the amplification at the center frequency. It has been shown that the lumped amplification process can be reasonably described by the distributed transfer function [27]

$$H(\Omega)=\frac{\ln[T(\Omega)]}{z_a}=\frac{G}{z_a}-\frac{1}{z_a}\ln\left[1+2i\frac{\Omega}{B}\right]$$
$$\approx G/z_a-2i\frac{\Omega}{Bz_a}-2\frac{\Omega^2}{B^2z_a}+i\frac{8}{3}\frac{\Omega^3}{B^3z_a}\ , \tag{18}$$

provided that $z_a=Z_a/Z_0\ll 1$. With α_l and $\Gamma=\alpha_l Z_0/2$ as the real and the normalized fiber loss coefficient, respectively, we may define a transfer function $H_f(\Omega)$ which accounts for the combined effect of distributed loss and amplification as

$$H_f(\Omega)=\delta-i\Delta\Omega-\beta\Omega^2+i\gamma_f\Omega^3\ . \tag{19}$$

Here, the net gain δ, the shift in the inverse group velocity Δ, the gain dispersion β and the third-order dispersion γ_f, caused by the finite amplifier bandwidth, are given by

$$\delta=\frac{G}{z_a}-\Gamma\ ,\quad \Delta=\frac{2}{Bz_a}\ ,\quad \beta=\frac{2}{B^2z_a}\ ,$$
$$\gamma_f=\frac{8}{3B^3z_a}=\tfrac{2}{3}\sqrt{2\beta^3z_a}\ . \tag{20}$$

By taking into account (19) the evolution of the pulse envelope $u(z,t)$ in a lossy fiber link with lumped amplifiers and $z_a\ll 1$ may now be described by the averaged perturbed nonlinear Schrödinger equation [38]

$$i\frac{\partial u}{\partial z}+\frac{1}{2}\frac{\partial^2 u}{\partial t^2}+|u|^2u=i\gamma\frac{\partial^3 u}{\partial t^3}+i\delta u+i\beta\frac{\partial^2 u}{\partial t^2}\ , \tag{21}$$

where Δ was removed by a Galilei transformation and γ reflects the TOD caused by the fiber and the filter ($\gamma+\gamma_f\to\gamma$). The envelope $u(x,t)$ is related to $\Psi(z,t)$ [see (1)] by

$$\Psi(z,t)=\left[\frac{2\Gamma z_a}{1-\exp(-2\Gamma z_a)}\right]^{1/2}\exp(-\Gamma\tilde{z})u(z,t)\ , \tag{22}$$

where $\tilde{z}$ is the distance in one amplifier span [53].

It has been shown that "averaged solitons" may exist provided that $\beta=3\delta$ [55]. In what follows we always use this condition. Moreover, for a fairly strong filter ($\beta=0.15$) and $z_a=0.1$ we can estimate the bandwidth-caused TOD to be $\gamma_f\approx 0.017$ being well below the threshold for the resonance radiation and much less than the fiber TOD near the zero-dispersion wavelength. Nevertheless, it shows that far from the ZDW filter-induced TOD may have some influence on the soliton propagation.

Because we are going to use the KSA below we have to specify the perturbation R that reads now as

$$R(\{u\})=i\delta u+i\beta\frac{\partial^2 u}{\partial t^2}\ , \tag{23}$$

where the trial function (5) holds now likewise for u.

Equations (8) and (9) remain unaltered whereas we get instead of (6) and (7) for the frequencies and the amplitudes

$$\frac{d\mu_n}{dz}=(-1)^n16\nu^3\exp(-2\nu r)\cos\Phi-\tfrac{16}{3}\beta\mu_n\nu_n^2\ , \tag{24}$$

$$\frac{d\nu_n}{dz}=(-1)^n16\nu^3\exp(-2\nu r)\sin\Phi$$
$$+2\delta\nu_n-8\beta\nu_n\left[\frac{\nu_n^2}{3}+\mu_n^2\right]\ . \tag{25}$$

We are now in the position to study the combined effect of TOD and BLA on the two-soliton interaction numerically by solving (21) or by the Karpman-Solov'ev perturbation approach using (8), (9), (24) and (25).

As mentioned in Sec. II TOD causes a decay of the soliton bound state where two solitons with opposite frequencies, and hence velocities, are left. This decay is advantageous for the two-soliton case because it prevents the coalescence of the pulses, but it is harmful for the N-soliton case in a transmission link because the soliton separate with distance and the capacity of the channel decreases. Recently, it was proposed to combine TOD and BLA [38,39]. The TOD induced frequency splitting may be exploited to prevent the collision in the first stage of propagation. Subsequently, the frequency splitting and the resulting pulse repulsion (see Fig. 2) may be suppressed by exploiting BLA which pulls the center frequency back to the center of the amplifier bandwidth ($\Omega=0$) [38]. However, the effect of BLA on the soliton propagation and interaction, if TOD is strong, has not been fully explained, to our knowledge.

In what follows we try to understand how BLA can affect the soliton propagation and interaction by manipulating the spectrum. In Sec. II we have shown that one prominent effect of a large TOD is the continuous generation of resonance radiation. This radiation has two important properties, namely, its frequency is offset from the soliton frequency inversely proportional to the TOD and its group velocity is close to that of the soliton. Thus, the conclusion is that one cannot separate this radiation from the soliton in time, but one has to cut it in the frequency domain. So, the question arises how this radiation is affected by BLA. Equation (19) tells us that two frequency domains exist. Frequency components with $|\Omega|<\Omega_0$ are amplified whereas those with $|\Omega|>\Omega_0$ are attenuated. The frequency $\Omega_0>0$ is given by $\mathrm{Re}[H_f(\Omega_0)]=\delta-\beta\Omega_0^2=0$ resulting in $\Omega_0^2=\delta/\beta$ or $\Omega_0^2=\frac{1}{3}$ if we use the condition $\beta=3\delta$. It is interesting that this frequency depends only on the net gain-bandwidth product rather than on the bandwidth directly. On the other hand, the resonance radiation has the frequency $\Omega_r\approx 1/2\gamma$, at least to the first order being sufficient here. Thus, the resonance radiation gets absorbed provided that $\gamma<\sqrt{3}/2\approx 0.866$ always being the case for realistic situations. This result is interesting because it implies that regardless of the bandwidth (at least for $\beta=3\delta$) the detrimental resonance radiation caused by TOD suffers an absorption due to the gain dispersion. This effect is displayed in Fig. 11. It can clearly be recognized that the resonance radiation generated by a fairly strong TOD ($\gamma=0.2$) is cut by the bandwidth-limited amplifiers regardless of the filter strength [$\beta=0.15$ in Fig. 11(b) and $\beta=0.03$ in Fig. 11(c)]. Of course, the strength of the absorption is proportional to the amplifier strength, as can be seen in Fig. 11, too. This behavior should have some positive implications for the soliton interaction. Note, however, that only the strong resonance radiation is cut completely but not the low frequency radiation caused by TOD. In Fig. 12 the evolution of two interacting pulses is shown. It is evident that the resonance radiation disappears and the positions of the pulses get stabilized provided that BLA acts [compare Fig. 4 with Figs. 12(a) and 12(b)]. But, it is clearly seen that strong amplifiers (large net gain, small bandwidth) strongly amplify the low-frequency radiation generated by TOD and the amplifiers. This is also displayed in Fig. 13 where the spectra of the different time slots are plotted (similarly as in Fig. 5 without BLA). A comparison with Fig. 5 shows both the cutting of the resonance radiation [see Fig. 13(a), 13(b), 13(d), and 13(e)], the attenuation of q_n [see

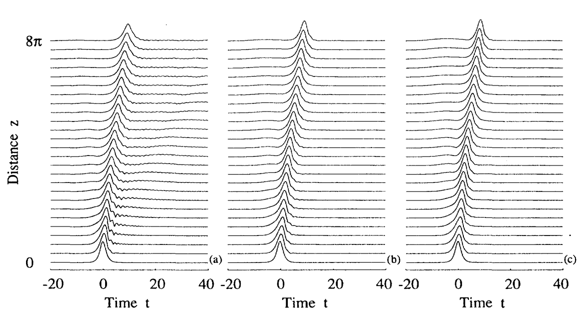

FIG. 11. Absorption of the resonance radiation by bandwidth-limited amplifiers; $\gamma=0.2$. (a) Without BLA; (b) with moderate BLA ($\delta=0.03,\beta=0.09$); (c) with strong BLA ($\delta=0.05,\beta=0.15$).

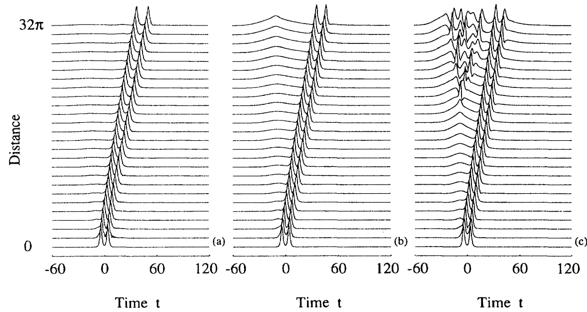

FIG. 12. Interaction of two solitons with large TOD ($\gamma=0.2$) and BLA: (a) weak BLA ($\delta=0.01,\beta=0.03$); partial stabilization of the positions as well as absorption of the resonance radiation; no amplification of low-frequency radiation; (b) moderate BLA ($\delta=0.03,\beta=0.09$); stabilization of positions and absorption of the resonance radiation; moderate amplification of low-frequency radiation; (c) strong BLA ($\delta=0.05,\beta=0.15$); stabilization of positions, but strong amplification of low-frequency radiation; evolving soliton instability.

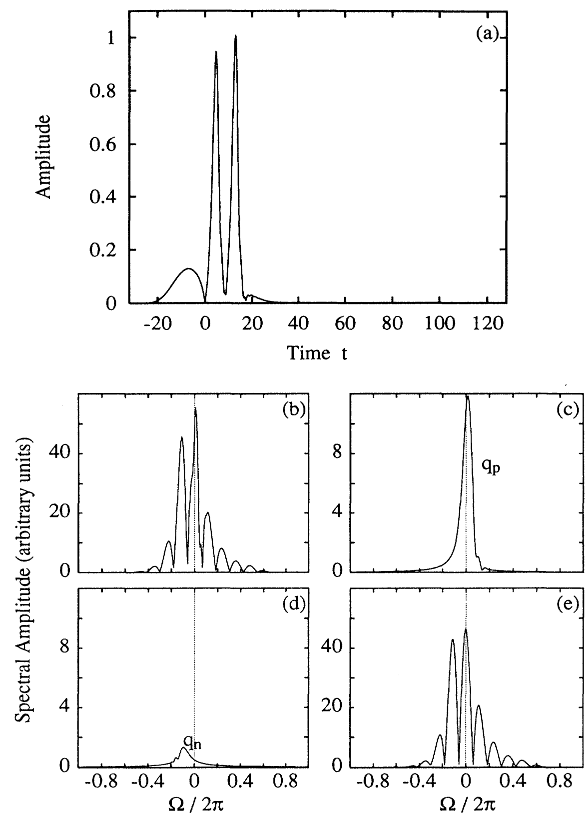

FIG. 13. Effect of BLA on the envelope and spectrum of two interacting pulses as well as of the dispersive waves at $z=8\pi$ and with $\gamma=0.2$ and $\delta=0.05,\beta=0.15$. (a) Envelope $|\Psi|$; (b) spectrum of the entire field shown in (a); absorption of the resonance radiation; (c) amplification of the low- and absorption of the high-frequency part of the spectrum of the dispersive waves in front of the leading edge ($t\leq 0.4$); (d) partial absorption of the spectrum of the dispersive waves behind the trailing edge ($t\geq 19.4$); absorption of the resonance radiation; (e) spectrum of the pulses and the dispersive waves in between them ($0.4<t<19.4$); absorption of the resonance radiation.

Fig. 13(d)] and the high-frequency parts of q_p and the amplification of the low-frequency part of q_p [see Fig. 13(c)]. Although the amplified low-frequency radiation separates from the solitons in time [see Fig. 12(a)] this radiation is detrimental as far as the N-soliton case is concerned rather than the two-soliton one. Because we have shown that the filtering effect survives even if we decrease dramatically the amplifier strength one solution could consist in the reduction of that amplifier strength. But this, in turn, has some negative implications on the suppression of the frequency splitting [see Fig. 12(a)]. Hence, for a too weak amplifier the pulses would separate. Thus, one has to look for a compromise. If we use a moderate amplifier strength we can almost prevent the amplification of the low-frequency radiation as well as the separation of the pulses as shown in Fig. 12(b). If this is not feasible in transmission links, because this would entail a reduction of the amplifier spacing, one can imagine that this low frequency radiation might be absorbed by using sliding filters [29] with a negative sliding rate, by replacing the linear gain partially by a nonlinear one [30] or by using nonlinear amplifying loop mirrors [57].

It was shown that sliding filters can cut the low-frequency radiation caused by the amplifiers [29]. Because the low-frequency radiation generated by TOD can hardly be distinguished from that radiation it should be absorbed likewise. The same applies for the nonlinear gain where the large amplitude components are preferred in the amplification process.

Moreover, as mentioned in Sec. II, the Karpman-Solov'ev perturbation approach fails if TOD gets strong and the resonance radiation comes into the play. Following this argument its applicability should considerably improve when that radiation disappears. So, it should be reasonable to describe the combined action of TOD and BLA with the KSA even when TOD is strong [39]. In fact, it can be shown that the stabilization of the separation of the pulses can be properly described (see Fig. 14 and compare with Fig. 7) by that perturbation approach. But in comparing the results with the numerical calculations it turns out that the absolute position of the pulses differs. This is not astonishing because KSA yields a velocity proportional to γ whereas it has been shown [38] that for strong TOD higher-order terms have to be taken into account [see (12)]. So, the conclusion is that KSA may be used to describe qualitatively correct the interaction of solitons for strong TOD provided that a moderate BLA acts and the amplification of the low-frequency radiation does not lead to soliton instability. This is due to the fairly good conservation of the pulse shapes and the absorption of the resonance radiation improving the reliability of the trial function (5). The quantitative correct description would require higher-order perturbation approaches as proposed by Elgin [47].

Eventually, we may conclude that even weak BLA may considerably improve the stability of the positions and the amplitudes of the solitons by pulling back the frequencies of the pulses to the center of the amplifier gain curve and by cutting the detrimental resonance radiation caused by strong TOD.

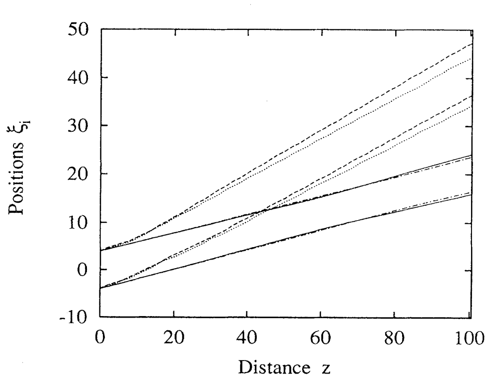

FIG. 14. Evolution of the positions of both pulses for different amplifier strengths—BPM vs KSA. Parameters: $r(0)=8$; $\nu_i-0.5$; $\mu_i=0$, $i=1,2$; $\gamma=0.2$. Moderate BLA ($\delta=0.03,\beta=0.09$): dashed line—BPM; solid line—KSA. Strong BLA ($\delta=0.05,\beta=0.15$): dotted line—BPM; dash-dotted line—KSA.

V. CONCLUSIONS

In this paper we have investigated the soliton interaction near the zero-dispersion wavelength. Particular emphasis was paid to the influence of the various radiation contributions being generated continuously by a TOD. It was shown that a TOD does not suffice to prevent the coalescence of N solitons. As far as TOD is weak (no resonance radiation) the positions and the amplitudes of solitons in a train can be stabilized by applying a proper phase modulation where we have derived the critical modulation amplitudes to achieve this effect. For a strong TOD where the resonance radiation may destroy the pulses bandwidth-limited amplification can be used to absorb this detrimental radiation. We have shown that this effect does only depend on the net gain-bandwidth product rather than on the bandwidth directly. Thus, the resonance radiation may be cut by very weak amplifiers provided that TOD attains realistic values ($\gamma<\sqrt{3}/2$). This effect of BLA may be used to stabilize the positions and the amplitudes of interacting solitons by pulling back the frequencies and cutting the resonance radiation. Moreover, the amplifier strength may be reduced to avoid the amplification of low-frequency radiation generated by the amplifiers and TOD. Although this radiation separates in time from the soliton as far as the two-soliton case is concerned it is detrimental in the N-soliton case. Eventually, the limits of applicability of a perturbation approach were identified near the zero-dispersion wavelength.

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APPENDIX: LIMITS OF APPLICABILITY OF THE PERTURBATION APPROACH

When TOD is taken into account the modified nonlinear Schrödinger Eq. (1) (with $R=0$) loses its integrability. Hence, one has to resort to numerical methods, as the beam propagation method (BPM) or to perturbation approaches where the Karpman-Solov'ev approach (KSA) [43] [see (5)–(9)] represents a very versatile and powerful variant. The main assumptions made to derive the relevant ordinary differential Eqs. (6)–(9) are as follows.

(a) Dispersive radiation can be neglected.
(b) The solitons keep their sech shape with the fixed relation between width and amplitude.
(c) The relative fluctuations in the amplitudes and the velocities are small.
(d) The separation between the pulse are not too small.

The essential parameters that can be varied are the strength of the TOD (γ), the initial separation $[r(0)]$ and the initial phase difference $[\delta(0)]$ where the interest is focused on in-phase $[\delta(0)=0]$ and out-of-phase $[\delta(0)=\pi]$ pulses. The aim is to identify conditions where KSA

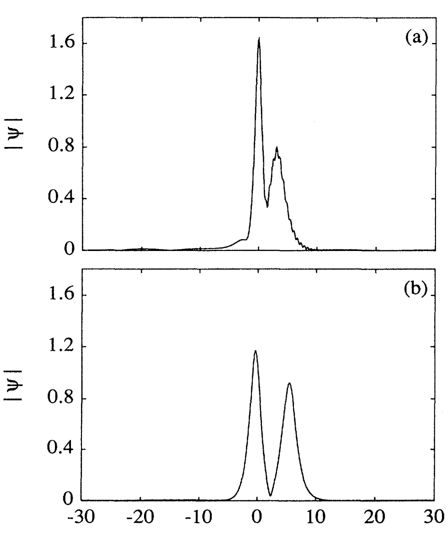

FIG. 15. Shapes of both interacting pulses at the point of minimum separation if TOD is moderate ($\gamma=0.05$): (a) $r(0)=6$; (b) $r(0)=8$.

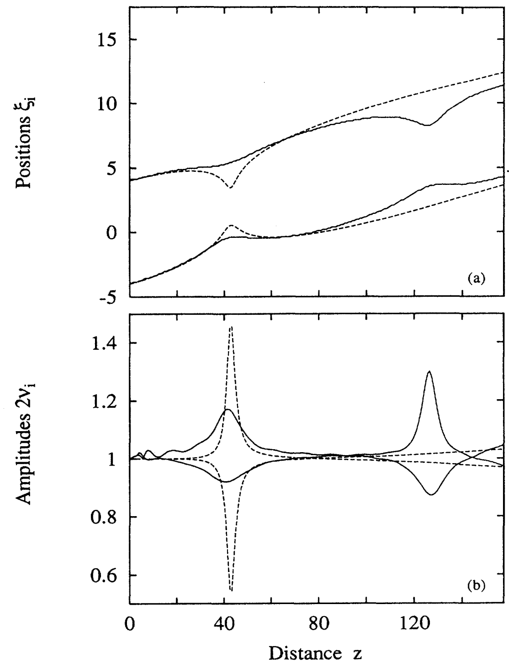

FIG. 16. KSA vs BPM. Evolution of the positions (a) as well as the amplitudes (b) of two in-phase pulses for moderate TOD($\gamma=0.05$) and moderate initial separation $[r(0)=8]$: solid lines—BPM; dashed lines—KSA.

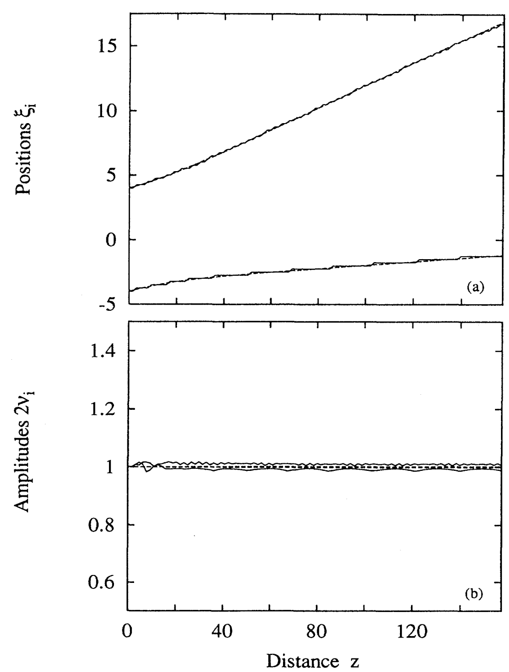

FIG. 17. KSA vs BPM. Evolution of the positions (a) as well as the amplitudes (b) of two out-of-phase pulses for moderate TOD ($\gamma=0.05$) and moderate initial separation $[r(0)=8]$: solid lines—BPM; dashed lines—KSA.

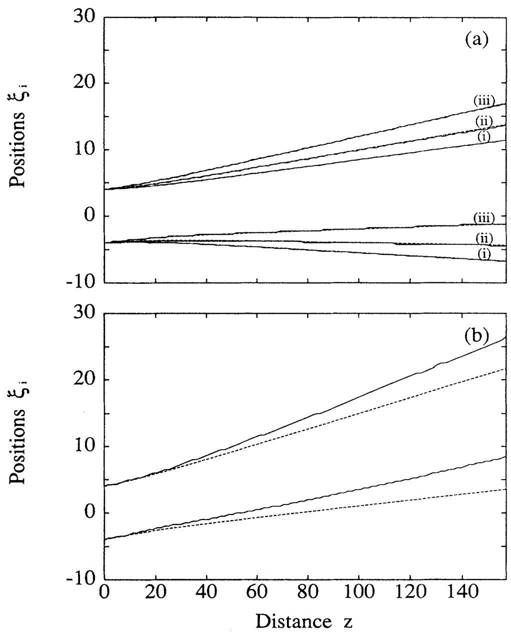

FIG. 18. KSA vs BPM. Evolution of the positions of two out-of-phase pulses for different strength of TOD as well as moderate initial separation $[r(0)=8]$. (a) Weak and moderate TOD: (i) $\gamma=0.015$, (ii) $\gamma=0.03$, (iii) $\gamma=0.05$. (b) Strong TOD: $\gamma=0.08$; solid lines—BPM; dashed lines—KSA.

works properly provided that TOD comes into the play. To this end we have compared the results of several BPM runs with the assumptions and the results provided by solving (6)–(9).

Assumption (a) requires that we are below or close to the threshold of resonance radiation, hence, we conclude that $\gamma \leq 0.05$ and restrict essentially to that case.

Then, we have changed the initial separation. In Fig. 15 the pulse shapes after a certain propagation distance are plotted for two typical examples. We may conclude that assumption (b) and (c) are violated for $r(0)=6$, but fairly well fulfilled for $r(0)=8$. Thus, a good estimation for the minimal separation should be for $r(0)\geq 8$. This is confirmed by Fig. 16 where we have plotted the fluctuations of the amplitudes and the positions as they result from BPM and KSA. The agreement is rather good as far as one is not at the position where the separation gets minimal. In Fig. 17 the evolution of the positions and the amplitudes is shown if one uses out-of-phase pulses $[\delta(0)=\pi]$. A perfect agreement can be recognized. To complete our studies we have checked whether we can increase the TOD even beyond the threshold of resonance radiation for out-of-phase pulses. The result is shown in Fig. 18. Obviously, both the resonance radiation as well as higher-order perturbation terms that have not been taken into account prevent a better agreement. That higher-order term might play an important role was already identified in inspecting Fig. 14 where the resonance radiation was cut by BLA.

Eventually, we can state that the Karpman-Solov'ev approach may be used to describe the two-soliton interaction near the zero-dispersion wavelength provided that the TOD is not too strong $(\gamma \leq 0.05)$ and the separation is not too small $[r(0)\geq 8]$ for any initial phase difference. Matching both conditions the agreement is almost perfect if out-of-phase pulses are used.

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